

Estimation and Asymptotic Inference in the First Order AR-ARCH Model

Stima e inferenza asintotica nel modello AR-ARCH di primo ordine

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Riassunto

This paper studies asymptotic properties of the quasi-maximum likelihood estimator (QMLE) and of a suggested modified version for the parameters in the first order autoregressive (AR) model with autoregressive conditional heteroskedastic (ARCH) errors.

The modified QMLE (MQMLE) is based on truncation of the likelihood function and is related to the recent so-called self-weighted QMLE in Ling (2006). We show that the MQMLE is asymptotically normal irrespectively of the existence of finite moments, as geometric ergodicity alone suffice. Moreover, our included simulations show that the MQMLE is remarkably well-behaved in small samples. On the other hand the ordinary QMLE, as is well-known, requires finite fourth order moments for asymptotic normality. But based on our considerations and simulations, we conjecture that in fact only geometric ergodicity and finite second order moments are needed for the QMLE to be asymptotically normal. Finally, geometric ergodicity for AR-ARCH processes is shown to hold under mild and classic conditions on the AR and ARCH processes.

Keywords: ARCH; Asymptotic theory; Geometric ergodicity; QMLE; Modified QMLE.